Bachelor Level / First Year/ Second Semester/ Science
Computer Science and Information Technology (MTH. 155)
(Linear Algebra)

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

Attempt all questions:

Group A

1. Write down the conditions for consistent of non-homogeneous system of linear equations.
2. What is meant by independent of vectors?
3. What is normal form of a matrix?
4. Define nonsingular linear transformations with suitable example.
5. Consider the matrix $A = \begin{pmatrix} 2 & 5 \\ 1 & 7 \end{pmatrix}$ as a linear mapping. Write the corresponding co-ordinate equations.
6. State the numerical importance of determinant calculation by row operation.
7. Show that $\{(1,1),(-1,0)\}$ form a basis for R^2.
8. Let $T : R^2 \to R^2$ be a linear transformation defined by

   $T(x, y) = (x + y, y)$. Find Ker T.
9. If λ is an eigen values of a matrix A, find the eigen values of A^{-1}.
10. Let u = (1, 2, -1, 3) and v = (3, 0, 2, -2). Compute the inner product $\langle u, v \rangle$.

Group B

11. Determine whether the following vectors in R^3 are linearly dependent:

    (a) (1, 0, 1), (1, 1, 0), (-1, 0, -1),

    (b) (2, 1, 1), (3, -2, 2), (-1, 2, -1).

12. Investigate and interpret geometrically the transformation of the unit square whose vertices are O(0, 0, 1), A(1, 0, 1), B(0, 1, 1) and C(1,1,1) effected by the 3×3 matrix

$$\begin{bmatrix} 1 & 1 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix}.$$

OR

Is the set of vectors $\{(1, 0, 1), (0, 1, 0), (-1, 0, 1)\}$ orthogonal? Obtain the corresponding orthonormal set in R^3.

13. In the vector space R^2, express the given vector (1, 2) as a linear combination of the vectors (1, -1) and (0, 1). Also find the coordinates of (1, 2) with respect to (1, -1) and (0, 1).

14. Find the matrix representation of the linear transformation $T : R^2 \to R^2$ defined by $T(x, y) = (x, x + 2y)$ relative to the basis (1, 0) and (1, 1).

15. Let u and v be nonzero vectors in R^n and the angle between then be θ. Then prove that

$$u \cdot v = \|u\| \, \|v\| \cos\theta,$$

where the symbols have their usual meanings.

Group C

(5×8=40)

16. Test for consistency and solve:

$$2x - 3y + 7z = 5$$
$$3x + y - 3z = 13$$
$$2x + 19y - 47z = 32$$

17. Let U and V be vector spaces over a field and assume that dim U = dim V. If $T : U \to V$ is a linear transformation, then prove that the following are equivalent;

(i) T is invertable

(ii) T is one-one and onto, and

(iii) T is non-singular.

OR

Verify that the set of matrices of the form $\begin{bmatrix} 0 & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & a_{32} & a_{33} \end{bmatrix}$ is a subspace of the vector space of 3×3 matrices.

18. Verify Cayley-Hamilton Theorem for matrix:

$$A = \begin{bmatrix} 6 & 2 & -1 \\ -6 & -1 & 2 \\ 7 & 2 & -2 \end{bmatrix}.$$

19. Diagonalize the matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -1 \\ 0 & 0 & 3 \end{bmatrix}$

OR

Compute the multiplication of partitioned matrices for

$$A = \left[\begin{array}{cc|ccc} 1 & 2 & 4 & 6 & 7 \\ 0 & 1 & 1 & 2 & 0 \\ \hline 0 & 2 & 3 & 6 & -1 \end{array}\right] \text{ and } B = \left[\begin{array}{ccc} 0 & -1 & 2 \\ 2 & 3 & 1 \\ \hline 1 & 4 & 5 \\ 2 & 2 & 0 \\ 0 & 7 & 6 \end{array}\right].$$

20. Find the equation $y = \beta_0 + \beta_1 x$ for the least squares line that best fits the data points (2, 0), (3, 4), (4, 10), (5, 16).